Indian Statistical Institute, Bangalore
B. Math (II)
First semester 2014-2015
Backpaper Examination : Statistics (I)

Date: 31-12-2015 **Maximum Score 80** **Duration: 3 Hours**

1. To establish a standard for parachute design, a researcher recorded the following fill times, in seconds, for 27 standard parachutes, obtained under controlled test conditions.

.59	.38	.47	.43	.44	.37	.43	.37	.27	.54	.39	.89	.48	.52
.51	.49	.38	.38	.23	.44	.40	.36	.33	.82	.51	.44	.37	

   (a) Make a stem and leaf plot of these data.
   (b) Find the sample mean $\overline{X}$.
   (c) Give formula for sample standard deviation S. Do not compute.
   (d) Find the sample median M.
   (e) Find $100p$-th percentiles for $p = 0.2$ and 0.8.
   (f) Find the first and third quartiles.
   (g) Draw the box plot and identify the outliers.
   (h) For trimming fraction 0.05 obtain the trimmed mean $\overline{X}_{\mathrm{T}}$.
   (i) Explain how to obtain the trimmed standard deviation S_{T}. Do not compute.
   (j) Between the box plot and the stem and leaf plot what do they tell us about the above data set? In very general terms what can you say about the population from which these data arrived?

[$4 + 2 + 2 + 2 + 4 + 4 + 5 + 3 + 3 + 4 = 33$]

2. The independent random variables $X_1,\ X_2, ..., X_n$ have common distribution specified by

$$P(X \leq x|\alpha, \beta) \quad = \left| \begin{array}{cl} 0 & if\ x < 0 \\ \left(\frac{x}{\beta}\right)^{\alpha} & if\ 0 \leq x \leq \beta \\ 1 & if\ x > \beta \end{array} \right.$$

where α, β are positive. It was found that the length of cuckoos' eggs found in hedge sparrow nests could be modelled with this distribution. Obtain *method of moments estimators* as well as *maximum likelihood estimators* for α, β.

[20]

[PTO]

3. Suppose you can draw a random sample from $U \sim uniform[0,1]$. Explain how you would draw observations on a random variable W that has $Beta\,(m,n)$ distribution, where m and n are positive integers.

[10]

4. Let X uniform$(0,1)$ and W uniform$(0,h)$, $h > 0$, be independent. Define $Y = X + W$. Find the joint distribution of X and Y. Draw the support of (X,Y). Let $(X_1,Y_1),(X_2,Y_2),\cdots,(X_n,Y_n)$ be a random sample from this bivariate distribution of (X,Y). Let $V = \frac{1}{n}\Sigma_{i=1}^n X_i$ and $Z = \frac{1}{n}\Sigma_{i=1}^n Y_i$. Find ρ_{VZ}, the correlation coefficient between V and Z. How does ρ_{VZ} vary with h?

$[6+6+4=16]$

5. A 6-sided die is thrown 60 times. The number of times it lands with 1, 2, 3, 4, 5 and 6 face up is 10, 5, 9, 8, 20 and 8, respectively. Test whether the die is unbiased at $\alpha = 0.05$? Also report the p value.

[15]